The transition to turbulence in the wake of a circular cylinder

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(Received 9 December 1963)

The position of the region of transition to turbulence and the manner in which turbulence develops are investigated using a hot-wire anemometer to study the character of the flow in the wake of a circular cylinder. In the range of Reynolds numbers greater than 200 in which turbulent motion is developed, the region of transition to turbulence moves towards the cylinder with increasing Reynolds number. The manner of transition to turbulence appears to undergo a basic change as the region of transition moves from the periodic wake into the region of flow immediately behind the cylinder where the separated layers have not rolled into vortices, that is, as the Reynolds number increases from Roshko's transition range to his irregular range. When transition occurs in the periodic wake it is a result of distortion due to large-scale three-dimensional effects. Turbulence, when it develops in the separated layers, is preceded by two-dimensional Tollmien–Schlichting waves which eventually degenerate to turbulence by the action of small-scale three-dimensionalities.

1. Introduction

There exists a definite range of Reynolds numbers in which it is generally agreed that the flow past a circular cylinder is laminar everywhere. At Reynolds numbers above this range the wake always degenerates to turbulence, the point of onset of turbulent motion moving towards the cylinder with increasing Reynolds number and reaching the separation point on the cylinder at the critical Reynolds number. The flow only becomes fully turbulent, that is loses all traces of periodicity, at an even higher Reynolds number. Our interest lies in the regime in which the onset of turbulence is moving towards the cylinder and in which the flow in the wake cannot be simply described as either laminar or turbulent. Throughout the paper R will be used to denote the Reynolds number, which is based on the cylinder diameter.

The periodic wake first appears at a Reynolds number of about 40. It is very stable and is easily detected as far as 100 diameters downstream according to Kovasznay (1949), who made an extensive investigation of the wake behind a circular cylinder at low Reynolds numbers. The vortices decay slowly throughout the length of the street due to viscous dissipation and no turbulent motion develops. This description fits the wake up to a Reynolds number of 150 with the possible exception of a small range near 90 investigated by Tritton (1959). Roshko (1953) described this regime of vortex production as the 'stable range'.

This is followed by what he called the 'transition range' extending to a Reynolds number of 300. In this range there are irregular bursts of velocity in the wake, the bursts becoming more violent as Reynolds number increases. The shedding frequency is difficult to determine. It has been suggested by Tritton (1959) and Hama (1957) that this change is caused by the onset of a three-dimensional instability. At Reynolds numbers between 300 and the critical Reynolds number of about 3×10^5, a definite shedding frequency returns. By measurements of the energy spectrum, Roshko found that throughout this range the wake becomes fully turbulent between 40 and 50 diameters downstream, all traces of periodicity having disappeared. Roshko's interpretation of the transition range was that it reflects a change from the vortex-shedding mechanism being stable to it being unstable. He also suggested that, in the irregular range, transition occurs in the separated boundary layer, all the vortices once formed being turbulent. He did nothing further towards investigating in detail the position and manner of transition. Indeed, the only precise determination of the point of transition known to the author was made by Schiller & Linke for $3500 < R < 8500$ as long ago as 1933. They found that in this range transition, or the beginning of the region of transition, moves from 1·4 to 0·7 diameters downstream of the point of separation on the cylinder. Completing the picture for the vortex-shedding range of Reynolds numbers, the sudden fall in the drag coefficient at the critical Reynolds number is a result of the laminar boundary layer separating, becoming turbulent immediately and reattaching again (Roshko 1961).

The two-dimensional linearized stability theory developed by Tollmien predicted the selective amplification of disturbing frequencies preceding transition to turbulence. Since Schubauer & Skramstad (1948) verified this theory experimentally in the boundary layer on a flat plate regular sinusoidal waves immediately before transition have been observed in other types of flow. Sato in a series of experiments has obtained good agreement with theory for their appearance in the separated layer behind a rearward facing step (Sato 1956), in a rectangular jet (Sato 1960) and in the wake behind a thin flat plate (Sato & Kuriki 1961). The last of these papers is particularly interesting, for the experiments suggest some regions of similarity between the sinusoidal fluctuations which appear in the wake behind a flat plate and the periodic fluctuations in the wake of a circular cylinder. It is now recognized that between the essentially two-dimensional Tollmien–Schlichting waves and turbulence proper three-dimensional effects play an important role in both separated and attached boundary layers (Sato & Kuriki 1961; Klebanoff, Tidstrom & Sargent 1962).

The aim of this paper is to consider the manner of transition and the position of the region of transition as the Reynolds number changes over the range in which turbulent motion develops after the fluid has separated from the cylinder.

The stable, transitional, and irregular vortices described by Roshko (1953) were observed in about the same Reynolds number range: 150–300 in Roshko's experiments, 200–400 in our low-turbulence wind tunnel. It was found that a basic change in the flow, as far as transition to turbulence is concerned, occurs at a Reynolds number of 400. Below this Reynolds number transition to turbulence proper occurs in the fully formed vortex street by means of low-frequency irregular fluctuations which are basically three-dimensional. At Reynolds numbers above 400, transition occurs in the separated layers *before the* vortices are formed and, once the transition region is well inside the formation region, it is preceded by the basically two-dimensional Tollmien–Schlichting waves.

2. Apparatus and method

The experiments were conducted in a return-circuit wind tunnel first described by Collis (1952). The working section of the tunnel had a 20 in. × 20 in. cross-section. The tunnel turbulence level was low; the longitudinal component of turbulence was less than 0·03 % at distances greater than 3 in. from the tunnel walls. It was possible to vary the speed continuously up to a maximum of 115 ft. sec^{-1}. An open-circuit tunnel, in which the longitudinal component of turbulent intensity was found to be about 1 %, was used to investigate the effect of free-stream turbulence on the extent of the stable range of Reynolds numbers. The cylinders spanned the tunnel vertically at the centre of the working section. They were smooth metal tubes of diameter 0·0305 in., 0·1 in., 0·125 in., 0·25 in. and 1 in. so that Reynolds numbers from 200 to 5×10^4 could be obtained. More than one cylinder could be used at each Reynolds number between 400 and 10,000.

The working section velocity was calculated from the stagnation and static pressures measured by pitot and static tubes mounted upstream of the cylinder but laterally displaced. As a check the shedding frequency was measured using a wave analyser and the velocity found from Roshko's relationship between the Strouhal number and the Reynolds number in the appropriate Reynolds number range. The fluctuating velocities were measured with hot-wire anemometers. These were made of Wollaston wire having a platinum core of 0·0001 in. diameter. The length of the etched portions varied between 0·6 and 1 mm. The wires were used on a constant-current bridge. The out-of-balance voltage was taken to the emitters of a pair of transistors arranged as a grounded-base amplifier. This in turn fed a pair of emitter followers balanced to give an overall gain of 30 in amplitude, 30 db in power. The output was observed on a cathode-ray oscilloscope. On all the photographs presented the velocity increases upwards.

Two types of probe were made. The one most frequently used consisted of a single wire which was held parallel to the cylinder axis and recorded the longitudinal component of the fluctuating velocity. The other was two electrically insulated parallel wires separated by less than 0·1 in. This enabled the signals from two adjacent points having the same downstream position to be viewed simultaneously on the two beams of the cathode-ray oscilloscope. The hot-wire holders were supported perpendicular to the cylinder axis at 30° to the flow direction from a point near the leading edge of a faired section which spanned the tunnel parallel to the cylinder. This was fitted on a railway so that the wires could be moved in the streamwise direction. The faired section could be moved across the stream by an electric motor.

The origin of co-ordinates was taken at the centre of the cylinder midway between the tunnel floor and roof. The x-axis was measured in the direction of the flow, the y-axis was perpendicular to the flow and the z-axis coincided with the cylinder axis. The x- and y-co-ordinates were determined by two microscopes mounted permanently outside the tunnel. One measured the x-co-ordinate directly and the other, which was far downstream of the cylinder, measured the y-co-ordinate indirectly through mirrors.

3. Presentation and discussion of results

Throughout this section turbulence will be defined as random fluctuations of high frequency imposed on the mean flow. By this is meant fluctuations with length scales which are small compared with the vortex size. Low-frequency irregularities are present at all Reynolds numbers above those corresponding to Roshko's stable range. These are velocity fluctuations with large length scale resulting probably from the three-dimensional character of the flow (Hama 1957). They are not included in our definition of turbulence at this stage but their origin needs further investigation and is discussed below.

Our aim was to find the 'point' of onset of the random high-frequency fluctuations. We were also interested in the appearance preceding transition of sinusoidal waves analogous to those encountered by, for example, Schubauer & Skramstad (1948) during experiments to study the laminar turbulent transition in the boundary layer on a flat plate. In their experiments it was found that, if the turbulent intensity of the stream is very low (0·03 %) or if the boundary layer is artificially excited, sinusoidal oscillations at a single frequency appeared a short distance ahead of the point of transition. Sato observed a similar phenomenon in the separated layer behind a rearward facing step (1956), in a rectangular jet (1960) and in the wake behind a thin flat plate (Sato & Kuriki 1961). Later experiments by Klebanoff *et al.* (1962) of the National Bureau of Standards have shown that there is a stage in the development of transition in the boundary layer on a flat plate when three-dimensional effects begin to dominate. The two-dimensional theory adequately predicts only the point of onset of instability. The three-dimensional theory which best explains experimental observations is that put forward by Benney & Lin (1960). A detailed study has not been made of the three-dimensional distortion of the regular sinusoidal oscillations we observed. The Reynolds number range which was studied has been divided into three regions to simplify discussion.

3.1. *The intermediate region*

There exists a range of Reynolds numbers ($1{\cdot}3\times10^3 < R < 8\times10^3$) in which transition is observed in the formation region, that is the region of flow inside the wake between the separation point on the body and the first appearance of the periodic vortex street. In each case the onset of turbulence is similar, the photographs at $R = 5{\cdot}2\times10^3$ (figure 1, plate 1) being typical. Some photographs show two traces at each hot-wire position. Directly downstream of the cylinder in the separated layer the flow is laminar and exhibits a dominant fundamental frequency even though the signal contains random components of a very much lower frequency. Downstream inside the wake the first turbulent bursts are waves at a single frequency imposed on the fundamental. This frequency is higher than that of vortex shedding and in the following discussion these oscillations will be called 'transition' waves. Further downstream, though still in the formation region, the fundamental frequency is almost masked and the flow degenerates to turbulence. The periodic wake with turbulent vortices is only apparent beyond this region. The signal observed in the periodic turbulent vortex street is illustrated in figure 1 (*f*), plate 1 and can be seen at all Reynolds numbers in and above this range up to the critical value. This shape is to be expected. Fluid from the laminar region outside the wake is drawn in between the vortices. Thus a wire which is not at the centre of the wake would experience both laminar and turbulent flow. The occasional 'spikes' on the signal at low velocity occur when the vortex centres on the opposite side of the street are unusually close to the wire.

In the intermediate range it was possible to examine the ratio of the frequency of the transition waves to the fundamental frequency by counting the number of transition waves imposed on one fundamental wavelength. It was only in the middle of this range that it was possible to photograph large regular transition waves on consecutive wavelengths of the fundamental frequency. At other Reynolds numbers within and above the intermediate range it was necessary to go outside the wake to photograph them. There then appeared to be high-frequency waves present but it was still difficult to obtain two adjacent wavelengths on which they were regularly and clearly imposed.

The edge of the wake must now be defined. It is generally thought of as the boundary between the rotational flow of the wake and the irrotational flow in the free stream, or as the point, moving from the centre of the wake, at which the mean velocity first equals the free-stream velocity. For simplicity, we decided on a definition enabling us to see clearly when the probe entered the wake. As the hot wire is moved from outside the wake towards the centre the amplitude of fluctuations at shedding frequency gradually increases: suddenly there is a jump in amplitude and, where low-frequency fluctuations can be seen inside the wake, this jump is accompanied by their appearance. At downstream stations where the low-frequency fluctuations are not seen, the fluctuations at shedding frequency take on the appearance shown in figure 1(*f*), plate 1. Farther from the axis the signal is sinusoidal. These changes mark the edge of the wake.

A graph of the number of transition waves per fundamental wavelength as a function of Reynolds number is shown in figure 2. The ratio can be seen to increase steadily from 2·5 at $R = 1{\cdot}3 \times 10^3$ to 8 at $R = 5 \times 10^3$ in the intermediate range. Two cylinder sizes were used in this range and thus the frequency of transition waves cannot be a function of vortex shedding frequency only. A few outside the range have been included illustrating a tendency for the ratio to increase as R increases, a value approaching 20 being indicated at $R = 2{\cdot}5 \times 10^4$.

In figure 3 the frequency of the transition waves is plotted against wind speed. For constant cylinder diameter the frequency is very nearly proportional to $U^{\frac{3}{2}}$. From figure 2 it can be seen that the ratio of the transition wave frequency to the fundamental frequency is proportional to $R^{\frac{1}{2}}$ at high Reynolds numbers. This can be justified by dimensional arguments. Let f_t be the frequency of the transition waves; this will be proportional to a characteristic velocity divided by a length. Assuming that the transition wave frequency is typical of the flow rather than the position of transition we let the velocity be U, the free stream

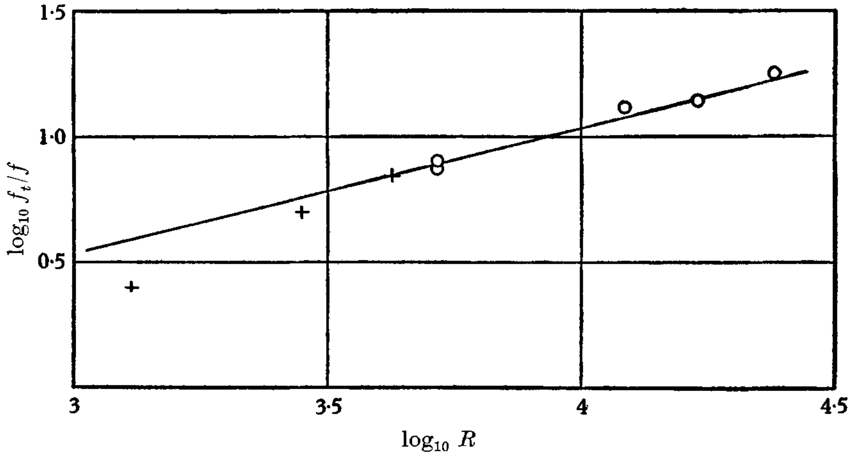

FIGURE 2. The ratio of transition wave frequency to fundamental frequency as a function of Reynolds number. ○, $d = 1$ in.; +, $d = \frac{1}{4}$ in.

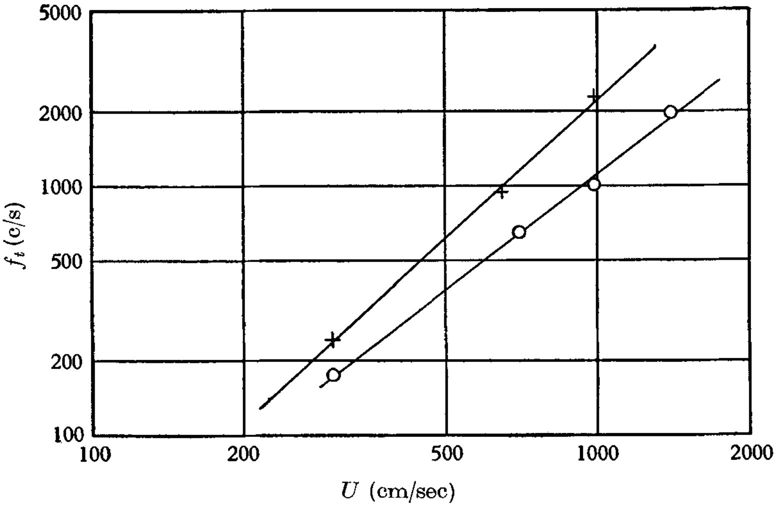

FIGURE 3. The frequency of transition waves plotted against free-stream velocity, U. ○, $d = 1$ in.; +, $d = \frac{1}{4}$ in.

velocity, and δ_1, the boundary-layer thickness at the point of separation, be the characteristic length. In a laminar boundary layer, δ is proportional to $(\nu l/U)^{\frac{1}{2}}$, where l is the distance along the surface. Therefore δ_1 is proportional to $(\nu l_1/U)^{\frac{1}{2}}$, where l_1 is the distance along the surface from the forward stagnation point to the point of separation on the cylinder and is proportional to the cylinder diameter. Thus

$$\delta_1 \propto (\nu d/U)^{\frac{1}{2}} \quad \text{and} \quad f_t \propto U^{\frac{3}{2}}/(\nu d)^{\frac{1}{2}}.$$

The fundamental shedding frequency, f, is characterized by the Strouhal number, S, where S equals fd/U and is found experimentally to be approximately equal to 0·2 for all Reynolds numbers above Roshko's stable range. Therefore

$$\frac{f_t}{f} \propto \frac{U^{\frac{3}{2}}}{(\nu d)^{\frac{1}{2}}} \frac{d}{U} = R^{\frac{1}{2}}.$$

It can be seen from the graph that the points obtained at lower Reynolds numbers fall away from this relationship. At these Reynolds numbers the region of transition is approaching the end of the formation region. When the waves appear far from the cylinder the flow is no longer characterized by conditions at the separation point. These separation-point conditions characterize a rectilinear shear layer. When the shear layer rolls up, the velocity gradient across the layer is reduced. Since the gradient is reduced, f_t is reduced below the $R^{\frac{1}{2}}$ dependence, as R decreases and transition approaches the end of the formation region. Sato found that the transition wave frequency was proportional to $U^{\frac{1}{2}}$ in the boundary layer separated from a rearward facing step (1956) and in the wake behind a thin flat plate (Sato & Kuriki 1961).

The beginning of the region of transition was marked by the appearance of transition waves since the flow degenerates to turbulence immediately afterwards. It moves towards the cylinder from 1·2 to 0·7 diameters downstream of the centre of the cylinder as R increases from 2×10^3 to $8{\cdot}5 \times 10^3$. These points are always inside the formation region, the periodic wake appearing 2 to 3 diameters downstream of the cylinder. The results agree reasonably well with those of Schiller & Linke who found that in the range $3500 < R < 8500$ the transition point, measured from the point of separation, moved from 1·4 to 0·7 diameters downstream.

That the ratio of the transition wave frequency to the fundamental frequency is a monotonically increasing function of R rules out the explanation that these waves are a spurious phenomenon resulting from vibration of the hot wire and its support. The same type of probe was used throughout the series of experiments. It seems probable that the waves are analogous to those observed by Sato. One difference is that his sinusoidal waves appeared to increase gradually downstream. This growth is not apparent inside the wake with our transition waves, possibly because turbulence develops so soon after their appearance. Our waves occur first in bursts as is evident from photographs of their appearance both inside and outside the wake. Outside the wake however something closer to Sato's gradual growth is observed. He detected also a region flanking the wake in which the fluctuations are always sinusoidal although the fluctuations near the axis may be irregular.

In their paper of 1961 Sato & Kuriki find similarities between transition waves behind a thin flat plate and vortex streets. They divide the transition region into three subregions:

(*a*) The linear region in which sinusoidal oscillations appear and grow exponentially downstream.

(*b*) The non-linear region in which the experimental results can be explained by a vortex model, consisting of an antisymmetrical double row of vortices.

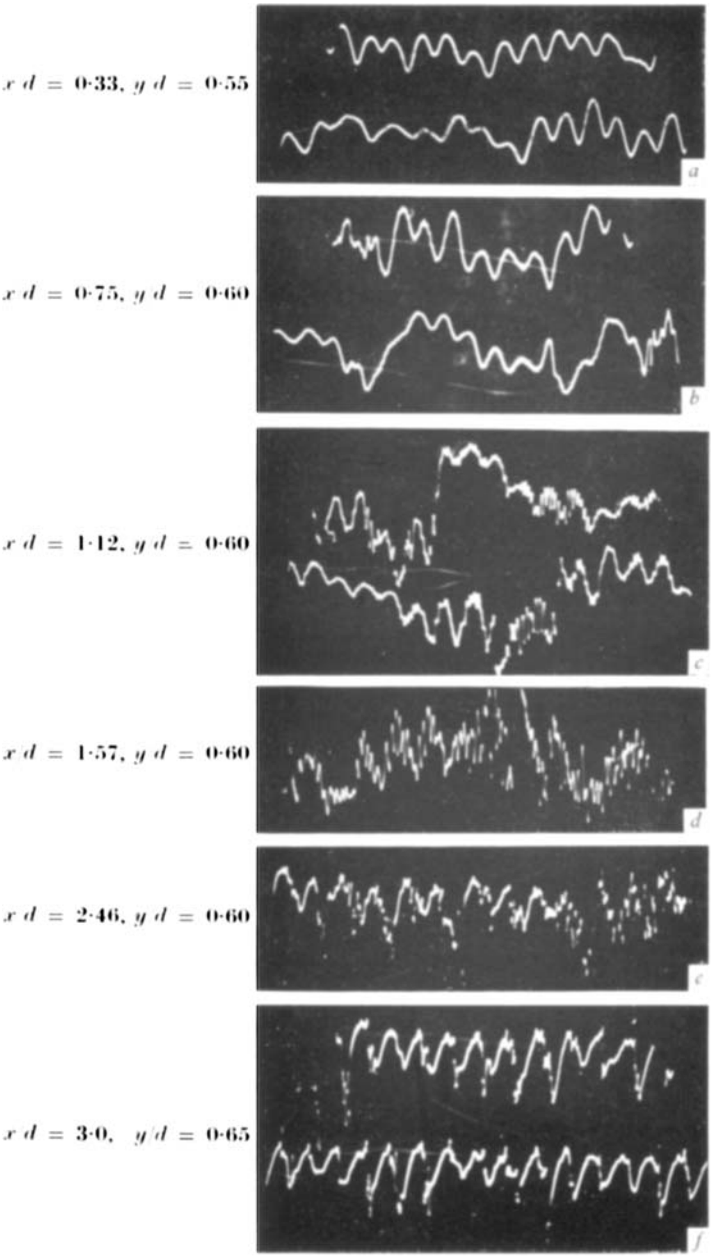

FIGURE 1. The development of turbulence in the intermediate range $R = 5{\cdot}2 \times 10^3$, $d = 1$ in. Some photographs show two traces at each hot-wire position.

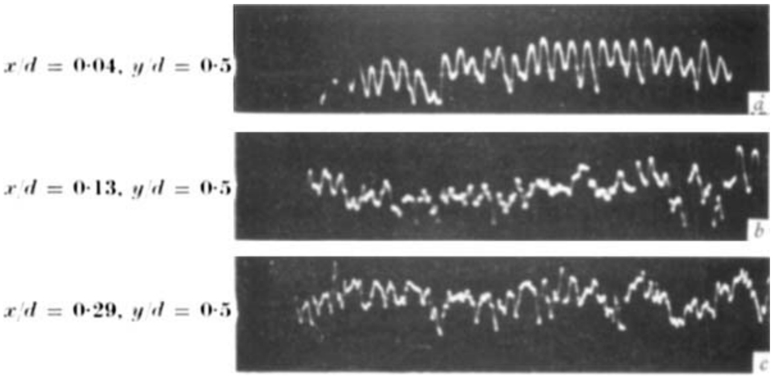

FIGURE 4. The development of turbulence at Reynolds numbers above the intermediate range. $R = 4{\cdot}5 \times 10^4$, $d = 1$ in.

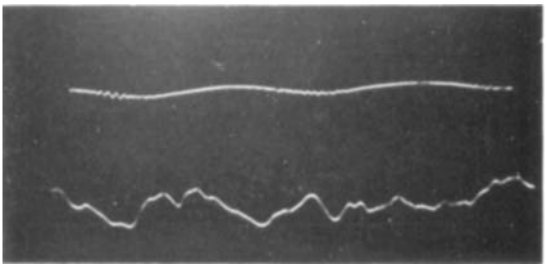

FIGURE 5. Comparison of the signals inside and outside the separated boundary layer using two hot wires simultaneously. $R = 2{\cdot}3 \times 10^4$, separation of hot wires = 1·5 mm, $x/d = 0{\cdot}46$. Outer wire (top trace): $y/d = 0{\cdot}58$.

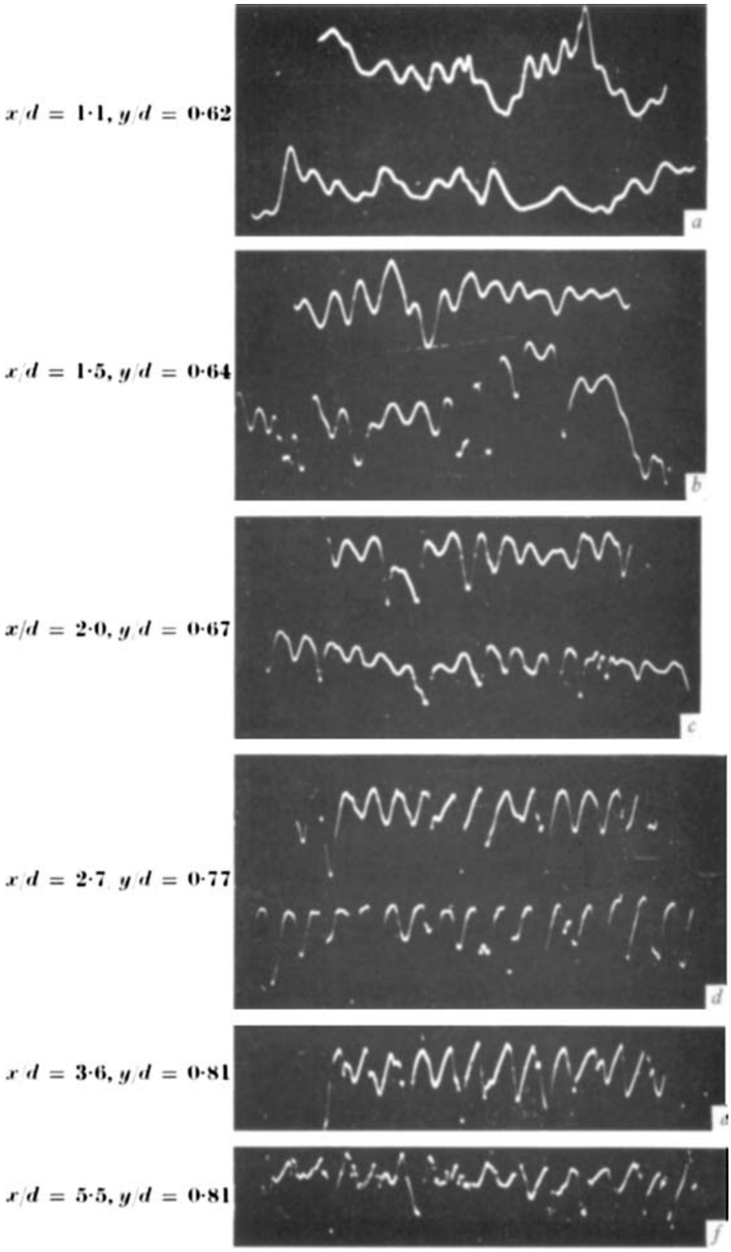

FIGURE 6. The development of turbulence at the lower end of the intermediate range. $R = 1\cdot35 \times 10^3$, $d = \frac{1}{4}$ in.

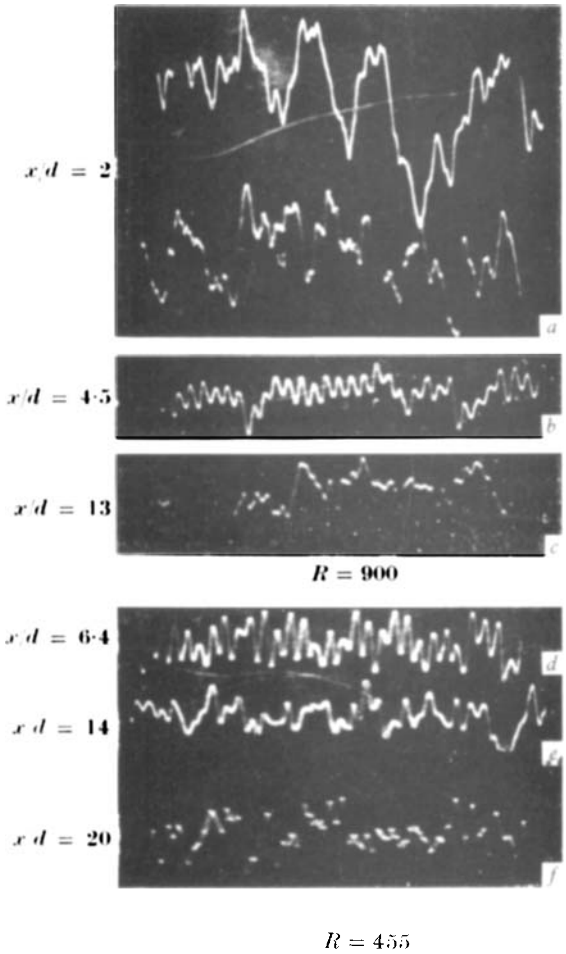

Figure 7. The downstream development of the wake in the transition range. $d = 0{\cdot}87$ mm.

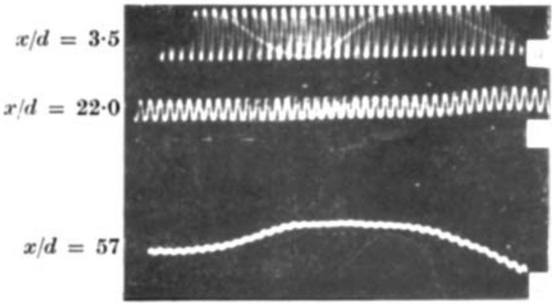

FIGURE 8. The downstream development of the wake in the stable range. R = 180, d = 0·87 mm.

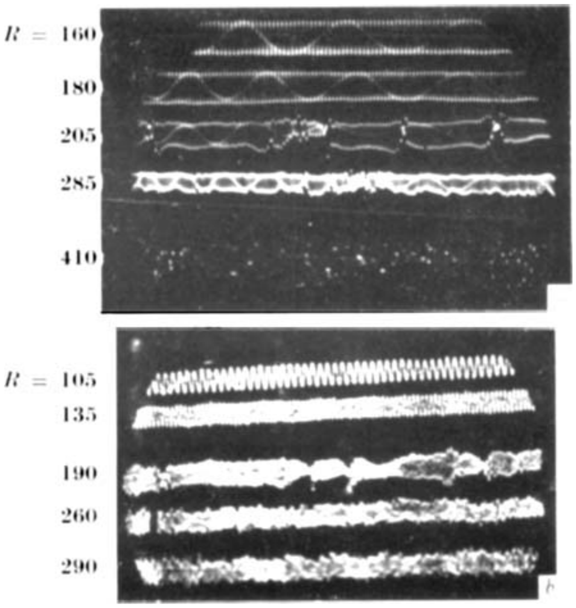

FIGURE 10. The development of the irregular modulation of the signal as the Reynolds number increases. Turbulence level: (*a*) 0·03 %, (*b*) 1 %; x/d = 8.

(*c*) The three-dimensional region in which the two-dimensional vortices are distorted by three-dimensionalities in the flow and the velocity fluctuation gradually develops into turbulence without bursts or breakdown.

The last two regions appear to correspond to those in the wake of a circular cylinder in Roshko's 'irregular' range, as Sato & Kuriki observed. Regions similar to the three described above are detected in the growth of transition waves only outside the wake in our experiment. They also observed that at low R the sinusoidal fluctuations behind the flat plate die out gradually without the development of turbulent flow. This suggests a correspondence with Roshko's 'stable' range, where the vortex street decays far downstream.

3.2. *Reynolds numbers above the intermediate range*

Considering now Reynolds numbers above the intermediate range the flows at Reynolds numbers of $1{\cdot}95 \times 10^4$, $2{\cdot}95 \times 10^4$ and $4{\cdot}5 \times 10^4$ were observed. In this region it is always possible to observe laminar periodic flow inside the separated boundary layer at the shoulder of the cylinder. This flow degenerates very suddenly to turbulence in less than one radius downstream of the cylinder centre. There then follows a period of completely turbulent motion until the periodic wake is formed downstream. Figure 4, plate 2, illustrates these changes.

At these Reynolds numbers transition waves are not visible inside the wake. The points on the graph were obtained with the hot wire outside the wake. Where transition is occurring in the wake the signal from an adjacent hot wire outside the wake clearly shows transition waves, as is seen from figure 5, plate 2. (Transition waves are visible outside the wake at lower Reynolds numbers where they can also be seen inside the wake.) The photograph is possibly misleading regarding the relative size of the fluctuations since the outer trace had ten times greater amplification. Amplifying the signal from the inner wire revealed no bursts of transition waves.

At all Reynolds numbers at which transition waves are visible they persist longer outside the wake than inside. Outside the wake fluctuations at shedding frequency are sinusoidal and low-frequency irregularities are not observed. Sinusoidal transition waves appear only at some of the regions of minimum velocity on the fluctuations at shedding frequency. Downstream they increase in amplitude and occur on each fundamental wavelength, later spreading to the regions of maximum velocity. Eventually they reach a maximum amplitude after which they become irregular.

Inside the formation region this growth is not detected. Turbulence develops almost immediately after the first appearance of transition waves. The first bursts of transition waves are detected slightly later inside than they are outside the wake.

The edge of the periodic wake is not as clearly defined as the edge of the formation region, the sudden increase in amplitude being largely due to spikes on the troughs (the position of minimum velocity). These spikes are due to the proximity of vortices on the opposite side of the street. Transition waves are only visible for a short distance downstream of the end of the formation region outside the wake. In this region they are obliterated on the troughs by the appearance of the spikes mentioned above. The transition waves originate in the formation region and are never seen to start beyond the end of the region. They are always irregular outside the periodic wake but appear unrelated to the turbulence within the vortices at the same downstream position.

At Reynolds numbers above the intermediate range it was possible to determine only a region of transition in which the flow changes from being laminar to fully turbulent. At a Reynolds number of $1{\cdot}95 \times 10^4$ this region occurs between 0·2 and 0·45 diameter downstream of the shoulder of the cylinder. The position of transition does not change noticeably between Reynolds numbers of $2{\cdot}95 \times 10^4$ and $4{\cdot}5 \times 10^4$ and in both cases the region of transition is between 0·02 and 0·29 diameter downstream. It is obvious that the point of transition does not move forwards appreciably from this region before the critical Reynolds number is reached.

3.3. *Reynolds numbers below the intermediate range*

At the lower end of the intermediate range the position of transition approaches the end of the formation region. The development of the wake was studied closely at a Reynolds number of $1{\cdot}3 \times 10^3$. At a downstream position of 1·5 diameter the flow is laminar and exhibits vortex-shedding though the amplitude is irregular and the signal contains jumps sometimes as large as twice the maximum amplitude of the fluctuations at shedding frequency. This behaviour continues until two diameters downstream where small turbulent bursts begin to appear, the low-frequency irregularities become less pronounced and occasional wavelengths of the shape typical of turbulent vortices appear. As the point of observation is moved downstream, the bursts become more frequent and the signal becomes that of a fully developed vortex sheet. Transition at this Reynolds number is shown in figure 6, plate 3. At the few points at which photographs of regular sinusoidal transition waves were observed the ratio of their frequency to the frequency of the fundamental appears to be about 2·5.

The region of transition extends about 3 diameters downstream and includes the beginning of the fully formed vortex sheet. The region of transition includes the end of the formation region down to $R = 400$. Turbulence develops in a manner similar to that described above for a Reynolds number of 1300. Small bursts of turbulence are observed before the end of the formation region. The bursts become more frequent farther downstream from the cylinder. At the beginning of the periodic wake, marked by the disappearance of the low-frequency irregularities characteristic of the formation region, every vortex consists of turbulent fluid. Below $R = 1300$, however, transition waves are not detected. It is thought that this is perhaps a result of their frequency being only slightly greater than the fundamental frequency, though the absence of irregularities outside the wake suggests this is not the reason.

At $R = 400$ it is difficult to decide whether the vortices are laminar or turbulent on formation. Some wavelengths are definitely laminar. The scale of the random fluctuations is not much less than the vortex size which may explain the difficulty referred to above. In order to discuss fully the transition to turbulence in the wake of a circular cylinder it is necessary to distinguish between transition to vortices composed of a turbulent fluid and transition to a turbulent wake in which the fundamental shedding frequency is undetectable. The definition of turbulence as random fluctuations of high frequency compared with the fundamental shedding frequency does not completely cover the transition to a turbulent wake far downstream. It does, however, allow an adequate discussion of transition, without consideration of low-frequency irregularities, at Reynolds numbers in the intermediate range and above. Here these irregularities are only observed in the formation region. Below a Reynolds number of 300 the vortices are all laminar on formation. The low-frequency irregularities are reduced at the end of the formation region as suddenly as at higher Reynolds numbers. They immediately begin to grow again, increasing downstream and eventually rendering the wake turbulent (i.e. randomly varying without trace of periodicity). Figure 7, plate 4, illustrates this sort of development.

The stable Karman vortex street is observed at R below 200. That no turbulent motion develops is seen from figure 8, plate 5, the decay of the wake downstream being a result of viscous dissipation.

3.4. *Low-frequency irregularities*

Transition in the wake of a circular cylinder is complicated by the appearance of low-frequency irregularities at all R above Roshko's stable range. At R between 200 and 400 low-frequency fluctuations are present everywhere in the wake, though they are considerably reduced at the end of the formation region. At higher R they are only clearly detectable in the separated layer before transition to turbulence.

One explanation, offered by Sato after experimental investigation, is that they are due to three-dimensionalities in the flow which would almost certainly be exhibited as low-frequency irregularities. Not much is known about three-dimensional effects though Hama (1957) conducted visualization experiments at R between 80 and 300. He found that below Roshko's 'transition' range there is no appreciable spanwise variation in the flow. The vortices are found in almost straight lines parallel to the cylinder axis. At R above the stable range an irregular transverse waviness appears close behind the cylinder. With increasing Reynolds number this becomes more violent. However at the beginning of Roshko's irregular range the transverse waves are more regular and periodic immediately behind the cylinder. As they go downstream they become turbulent. The last part of this description is applicable to our observations at $200 < R < 400$, i.e. that low-frequency irregularities become more vigorous downstream and eventually render the wake turbulent.

More work is required to investigate three-dimensional effects at higher Reynolds numbers. The only published work is a few measurements of the correlation length along the span of the cylinder (Prendergast 1958; El Baroudi 1960), Macovsky's (1958) demonstration of three-dimensionalities, and the work of Humphreys (1960) at the critical Reynolds number. One would expect three-dimensional effects to be present at all R above the stable range. The disappearance of low-frequency irregularities at R in and above the intermediate range after the flow has become turbulent in the formation region is puzzling. Were it not for the fact that the low-frequency irregularities are reduced at the end of the formation region even when laminar vortices are formed, the disappearance of irregularities after transition to turbulence in the formation region at higher R could be explained as a natural consequence of the transfer of energy from large-scale motions to motions of smaller scale in turbulent flow. This is the explanation for the eventual disappearance of periodicity downstream of the formation of the turbulent periodic wake.

3.5. *The length of the formation region*

An investigation was conducted to study how the length of the formation region varies with Reynolds number. It was decided that the beginning of the periodic wake was marked by the sudden reduction of the low-frequency irregularities always observed in the formation region. This definition agrees well with that based on the appearance of 'typical' turbulent vortices. The results are illustrated in figure 9. The length of the formation region, measured from the centre of the cylinder, appears to increase from two diameters at the beginning of Roshko's transition range to 2·5 diameters at the lower end of our intermediate range. Above this there is a steady fall in the length of the formation region with increasing Reynolds number. In this region it can be seen that there is a definite divergence between the results obtained with a $\frac{1}{4}$ in. cylinder and those obtained with a 1 in. cylinder. A similar dependence on cylinder diameter at these Reynolds numbers was noted when the steady pressure at 180° from the forward stagnation point on the cylinder was measured (unpublished work by J. H.

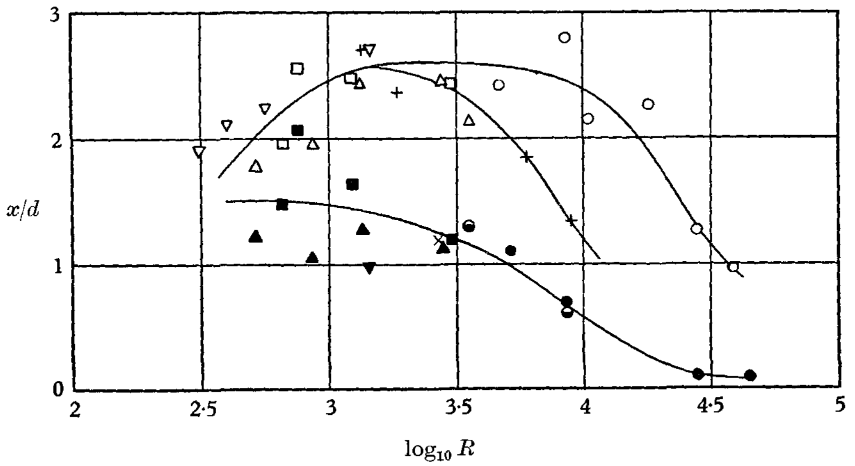

FIGURE 9. The lengths of the formation region and the region of laminar flow (measured from the centre of the cylinder). The formation region: ○, $d = 1$ in.; +, $d = \frac{1}{4}$ in.; △, $d = \frac{1}{10}$ in.; □, $d = \frac{1}{8}$ in.; ▽, $d = \frac{1}{16}$ in. The region of laminar flow: ●, $d = 1$ in.; ×, $d = \frac{1}{4}$ in.; ▲, $d = \frac{1}{10}$ in.; ■, $d = \frac{1}{8}$ in.; ▼, $d = \frac{1}{16}$ in., ⊖, Schiller & Linke (1933) (assuming the point of separation on the cylinder is 80° from the forward stagnation point).

Gerrard). It seems likely that the length of the formation region will affect the pressure at the rear of the cylinder. The dependence of pressure on cylinder diameter was observed only when the free-stream turbulence was low.

3.6. *Reynolds numbers in the stable range: the effect of free-stream turbulence*

During the investigations it was found that the stable range extended consistently to a Reynolds number very close to 200. It was, therefore, decided to carry out a brief study into the effect of free-stream turbulence on the extent of the range. An open-circuit tunnel of turbulence level 1 % and the return circuit tunnel, turbulence level 0·03 %, were used in the comparison and photographs illustrating the results are shown in figure 10, plate 5. From these photographs it is clear that in the low-turbulence tunnel the stable range extends to a Reynolds number above 180, irregular modulation of the signal first being apparent on the signal at $R = 205$. In the open-circuit tunnel there is evidence of slight regular modulation on the traces at $R = 105$ and $R = 135$. Nevertheless, the irregularity of amplitude, characteristic of the transitional range, is not apparent. In figure 10 the stable range ends therefore in the region $135 < R < 190$. Throughout the series of observations the stable range always terminated at some R between 160 and 200, the higher number always applying to the low-turbulence tunnel.

This verifies Roshko's suggestion that the turbulence level of the tunnel is an important factor in the extent of the stable range. However Roshko found that the stable range extended only to $R = 150$ in an open-circuit tunnel with turbulence level 0·03 % (i.e. the same as our return-circuit tunnel).

3.7. *Three-dimensional effects in transition*

The similarity between the development of the cylinder wake at $200 < R < 400$ and that of the separated layer before rolling up deserves further mention. In the wake at these R one observes laminar vortices subject to three-dimensional disturbances which are exhibited as low-frequency fluctuations which eventually render the wake turbulent, the vortex-shedding frequency disappearing. In the separated layer one observes transition waves which are possibly laminar vortices (see below). It is widely agreed that the next stage in transition is the onset of three-dimensionalities which distort the flow and are of prime importance in the development of turbulence.

A paper by Pierce (1961) presents photographs of the formation and growth of vortex sheets behind plates accelerated from rest. They show the regular formation of centres of vorticity within the vortex sheet before and after it rolls up. We suggest the same sort of phenomenon may be responsible for the transition waves we detected.

That three-dimensionalities play an important role in the development of turbulence has been adequately demonstrated experimentally (e.g. Klebanoff *et al.* 1962) and theories including non-linear terms which agree with experimental observations have been developed. Klebanoff *et al.* comment that, with a few qualifications, the theory of Benney & Lin (1960) most clearly describes what they observe. Benney & Lin's theory deals with transition in a shear flow, whilst Klebanoff *et al.* investigate transition in the boundary layer on a flat plate. The more detailed analysis by Benney (1961) of the non-linear interaction between the two-dimensional Tollmien–Schlichting wave and a superimposed three-dimensional wave having periodic spanwise variation in wave amplitude stresses the importance of the secondary vorticity generated in the downstream direction and predicts the presence, before actual breakdown, of longitudinal vortices of opposite sign above and below the critical layer, a region near $0{\cdot}2\delta$ (δ is the boundary-layer thickness) where the velocity of the primary wave is equal to the mean velocity (Klebanoff *et al.*)

There is, however, a recently published theory by Hama (1963) which, he suggests, gives a more reasonable account of the observations of Klebanoff *et al.* He predicts that when the two-dimensional vortex filament is perturbed by three-dimensional distortion it deforms progressively by its own induction. The vortex filament is elongated in the flow direction and the tip of the vortex is lifted up while the tail is pushed down (as Hama observed in an earlier experimental paper, 1957). The flow acquires a longitudinal vorticity component which Hama suggests is the way the longitudinal vortex system of Klebanoff *et al.* is formed.

There is still a gap in our knowledge of how deformation proceeds and of the ultimate cause of breakdown to turbulence.

4. Conclusions

The position and mode of transition to turbulence in the wakes of circular cylinders were investigated with a hot-wire anemometer. Stable, transitional and irregular ranges similar to those of Roshko (1953) were observed. Below $R = 200$ turbulent motion is not generated. In the range $200 < R < 400$, which corresponds to Roshko's transition range, the downstream development of turbulence is thought to be due to three-dimensional distortion. Above $R = 400$ transition occurs before the separated layer rolls up, the vortices once formed being turbulent. Regular sinusoidal waves appear to be a phenomenon connected with transition only when it occurs well within the region of vortex formation. These transition waves are observed at all Reynolds numbers above 1300.

The ratio of the frequency of the waves to the fundamental frequency is found to increase with R (figure 2) and the frequency of the waves for constant cylinder diameter is proportional to $U^{\frac{3}{2}}$ at high Reynolds numbers. This dependence suggests that the waves can be identified with the Tollmien–Schlichting waves observed by Sato.

In view of the fact that Tollmien (1929) accounted for the appearance of regular sinusoidal oscillations preceding transition quite adequately by a two-dimensional theory and that the wake is definitely two-dimensional in the stable range (except possibly at $R = 90$) it seems probable that:

(i) The instability producing the Karman vortex sheet is a two-dimensional one.

(ii) The instability responsible for transition waves, which may be identified with Tollmien–Schlichting waves, is basically two dimensional.

(iii) The low frequency oscillations at Reynolds numbers between 200 and 400 are associated with the three-dimensional character of the wake and lead to a distortion which eventually renders the wake turbulent.

Thus transition to turbulence occurs in two ways: (*a*) the amplification of a two-dimensional instability producing Tollmien–Schlichting waves and eventually leading (through small scale three-dimensionalities) to turbulence, (*b*) distortion by large-scale three-dimensionalities in the flow. The former occurs only when transition to turbulence takes place in the separated layers before they roll into vortices and the latter is the mechanism of transition when the vortices are laminar on formation but transition occurs downstream.

Though remaining in the periodic wake the region of transition moves towards the cylinder as R increases in the transition range ($200 < R < 400$). At $R = 400$ the vortices, once formed, are turbulent. There exists, however, no noticeable movement towards the cylinder until $R = 1300$ when the region of laminar flow begins to decrease from its length at that Reynolds number of about 1·4 diameters. The region of transition has almost reached the shoulder of the cylinder at a Reynolds number of about 5×10^4.

The length of the formation region increases from 2 to 2·7 diameters as R increases from 400 to 1300. It then follows the same sort of decrease as the region of laminar flow, the periodic wake being formed 1 diameter behind the cylinder at the highest Reynolds number.

Whilst it has long been known that the region of transition moves towards the cylinder as R increases, details of this movement have not previously been published. A study of the manner of transition was also thought necessary. Since it was observed that Tollmien–Schlichting waves occur in the boundary layer separated from a cylinder, this type of flow may now be added to those in which Tollmien–Schlichting waves are known to play an important part in transition to turbulence.

The author would like to thank Dr J. H. Gerrard for his continued help and interest during the preparation of this paper. Throughout the period of work the author was in receipt of a grant from the Department of Scientific and Industrial Research.

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